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stituents admit of the permutations of some transitive group of degree m_1. The constituent transitive groups are clearly of degree $\frac{n}{m_1}$ unless $G_1=1$. The last case does not need consideration when the order of the transitive group of degree m_1 is not a multiple of n.

[To be Continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from December Number.]

SCHOLION IV: *In which is expounded on a figure a certain consideration on which Euclid probably thought, in order to establish that Postulate of his as 'per se' evident.*

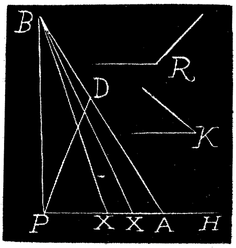

Fig. 12.

I premise first: within any acute angle BAX (Fig. 12.) can be drawn from any point X of AX a certain straight XB, which under designated even if obtuse angle R, which only with this acute BAX falls short of two right angles; a certain XB, say I, can be drawn, which at a finite remove meets this AB in a certain point B. For just that I have demonstrated in a Scholion after P. XIII. I premise secondly: these AB, AX (Fig. 25) can be understood as produced into the infinite even to certain points Y, and Z; and likewise the aforesaid XB (into the infinite and itself produced even to a certain point Y) can be understood to be so moved above this AB toward the parts of the point Z, that the angle at the point X toward the parts of the point A is always equal to the certain given obtuse angle R.

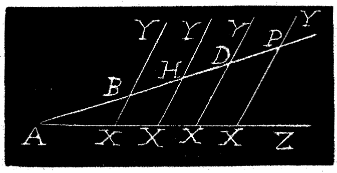

Fig. 25.

I premise thirdly: that Euclidean Postulate would be liable now to no doubt, if the aforesaid XY in this however great motion above the straight AZ cuts always that AY in certain points B, D, H, P, and so successively in other points more remote from this point A.

The reason is evident; since thus any two straights AB, XH lying in the same plane, upon which any straight AX cutting makes two angles toward the same parts *BAX*, *HXA*, less than two right angles, must at length meet toward those parts in one and the same point *H*.

I premise fourthly : likewise will be no doubt over the truth of the preceding hypothetical assumption, if those later external angles *YHD*, *YDP* and so any other succeeding ones, either always are equal to the preceding external angle *YBD*, or at least always will be not so much less but that any one of them always will be greater than any little designated acute angle *K*. For, this holding, it is manifest that this *XY* in that however great motion of its toward the parts of the point *Z*, never will cease to cut the aforesaid *AY*; which assuredly (from the preceding note) is sufficient for establishing the controverted postulate.

Solely therefore remains, that a certain adversary may say those external angles at greater and greater distance from that point *A* may become always less without any determinate limit.

But thence would follow, that that *XY* in that motion of its above the straight *AZ* would at length meet *AY* in a certain point *P* without any angle with the segment *PY*, so that indeed a segment of the two straights *APY*, and *XPY* would be in this way common.

But this is evidently repugnant to the nature of the straight line. [The possibility that *P* may be a point at infinity is here overlooked.]

But if indeed to anyone may seem less opportune the obtuse angle at that point *X* toward the parts of the point *A*, it may easily be supposed right ; so that indeed (in the motion of the aforesaid *XY* at angles always right above the straight *AZ*) more manifestly may appear that the single points of that *XY* are always moved equably relatively to the basal *AZ*; and therefore the aforesaid *XY* cannot go over from a secant into a non-secant of the other indefinite *AY*, unless either once in some point it precisely touches it, or meets it in some point *P*, where it has with this *AY* a common segment *PY*; each of which I will show contrary to the nature of the straight line in P. XXXIII.

Therefore in accordance with the true idea of the straight line, must that *XY*, in however great distance of the point *X* from the point *A*, always meet in some point this *AY*. And that this indeed (however small is supposed the acute angle at the point *A*) is sufficient for demonstrating, against the hypothesis of acute angle, the Euclidean Postulate, will follow from P. XXVII.

[To be Continued.]

THE BOND PROBLEM.

By J. K. ELLWOOD, A. M., Colfax School, Pittsburg, Pennsylvania.

What should an investor pay for one 7 per cent. $100. bond to run 20 years, interest payable semi-annually, in order to realize 8 per cent. per annum, payable semi-annually ?